Abstract:
Every power-law dynamical system used in chemistry and biology can be realized as a reaction network with (generalized) mass-action kinetics. The study of positive equilibria in such systems results in systems of generalized polynomial equations with real exponents and positive parameters. In this talk, we first describe our results on positive equilibria that are determined by the underlying graph (complex-balanced equilibria) of a generalized mass-action system. In particular, we discuss an effective characterization of the unique existence (in every invariant set and for all rate constants) using sign vectors of subspaces arising from the stoichiometric coefficients and the kinetic orders (exponents).

In the second part of the talk, we outline a recent result for arbitrary systems of generalized polynomial equations for positive variables, involving a positive parameter for each monomial. It turns out that the relevant geometric objects of the problem are a polytope arising from the coefficients and two subspaces representing differences and dependencies of the exponents. The dimension 'd' of this latter subspace, which we refer to as the monomial dependency, is crucial. In our framework, which is based on linear algebra and convex geometry, we rewrite the problem in terms of 'd' binomial equations on the coefficient polytope, involving 'd' monomials in the positive parameters. We illustrate our approach with applications to fewnomial theory.