

Abstract:

System and Control theory provide us with the easily understandable notions of controllable or autonomous system. Algebraically they correspond to (torsion-)free and torsion submodules of a system module.

Generally, the localization is a systematic way to create bigger rings (up to the ring of fractions) from a given one by inverting some sets of ring's elements. Localization applies naturally to ideals and modules over these rings. In algebraic analysis, we recognize classical coefficient rings of algebras of operators as localizations of polynomials or of power series. This provides a functorial connection between modules over considered rings, which can be used to explain the phenomenon of torsion. Due to my recent work, generalized  $S$ -torsion can be characterized as the subset of a given module, which vanishes after localizing the given ring at a denominator set  $S$ . In the case of non-commutative algebras of operators, which are domains,  $S$  has to be a multiplicatively closed Ore set. With J. Hoffmann we have found the canonical form for such sets, which helps to prove isomorphy of localized rings. We have also approached the algorithmic treatment of three types of localizations in two articles, and provided sophisticated implementations for these algorithms. For a localized module  $L$  there exists the unique corresponding module  $M$  over the unlocalized ring. The passage from  $L$  to  $M$  is called the Weyl closure of  $L$ ; its origins were in the algebra of linear partial differential operators, but nowadays the notion of Weyl closure applies to the mentioned process over arbitrary rings. It is a very important computation, widely used by other algorithms; it has been advanced further by many authors, among other by those from Linz. As of today, the algorithmic approach to Weyl closure is rather limited, covering the very basic (though already instrumental in applications) situations.