Abstract:
Tropical geometry is a combinatorial shadow of algebraic geometry, transforming polynomials into piecewise linear functions and their solutions (varieties) into polyhedral fans. This transformation is intricately linked to the concepts of Gröbner bases, originally pioneered by Buchberger, which provide a powerful tool in computational algebra. Specifically, all possible Gröbner bases of an ideal are encoded within a polyhedral fan, containing the tropical variety as a subfan. Despite its significance, the computational complexity associated with tropical varieties often confines practical computations to small-scale instances. In this talk, we introduce a geometric approach, enabling effective computation of various points within tropical varieties by facilitating Gröbner walks within the tropical fan. One application of this method is the computation of toric degenerations, which are popular objects in algebraic geometry, as they can be modeled on polytopes and there is a dictionary between their geometric properties and the combinatorial invariants of their polytopes. This dictionary can be extended from toric varieties to arbitrary varieties through toric degenerations. Finally, we will discuss some applications outside of mathematics; for example, in phylogenetic tree space and tropical data science.